Georgia Mathematics

Linear Algebra with Computer Science Applications

Mathematical Practices

0 Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals. LACS.MP

- 0.1 Make sense of problems and persevere in solving them. LACS.MP.1
- 0.2 Reason abstractly and quantitatively. LACS.MP.2
- 0.3 Construct viable arguments and critique the reasoning of others. LACS.MP.3
- 0.4 Model with mathematics. LACS.MP.4
- 0.5 Use appropriate tools strategically. LACS.MP.5
- 0.6 Attend to precision. LACS.MP.6
- 0.7 Look for and make use of structure. LACS.MP.7
- 0.8 Look for and express regularity in repeated reasoning. LACS.MP.8

Mathematical Modeling

1 Apply mathematics to real-life situations; model real-life phenomena using mathematics. LACS.MM.1

- 1.1 Apply mathematics to real-life situations; model real-life phenomena using mathematics. LACS.MM.1.1
- 1.2 Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts. LACS.MM.1.2
- 1.3 Using abstract and quantitative reasoning, make decisions about information and data from a contextual situation. LACS.MM.1.3
- 1.4 Use various mathematical representations and structures with this information to represent and solve real-life problems. LACS.MM.1.4

Abstract & Digital Reasoning

2 Investigate and describe real-life problems in linear algebra using an object-oriented programming language. LACS.ADR.2

- 2.1 Utilize sets, lists, dictionaries, indexing, and tuples in programming languages. LACS.ADR.2.1
- 2.2 Show and explain how to program and apply modules and control statements in programming languages. LACS.ADR.2.2
- 2.3 Program input and output features to read from and write to files in a programming assignment. LACS.ADR.2.3

Geometric & Spatial Reasoning

3 Solve contextual, mathematical problems involving vectors to explain real-life phenomena. LACS.GSR.3

- 3.1 Use coordinates to represent points in n dimensions and define and use arithmetic operations on n-dimensional points. LACS.GSR.3.1
- 3.2 Use vectors to find and interpret geometrical relationships between points in two and three dimensions, such as distance, and generalize these relationships to higher dimensions using n-dimensional vectors. LACS.GSR.3.2
- 3.3 Interpret adding, scaling, and linear combinations of vectors geometrically and algebraically. LACS.GSR.3.3
- 3.4 Find and use the dot product of two n-dimensional vectors. LACS.GSR.3.4
- 3.5 Use properties of the dot product to prove statements about vectors and to solve problems in context. LACS.GSR.3.5
- 3.6 Use the triangle inequality in n-dimensions. LACS.GSR.3.6
- 3.7 Find and use the cross product of two 3-dimensional vectors. LACS.GSR.3.7
- 3.8 Represent and perform vector operations using programming language classes that define the use of vectors. LACS.GSR.3.8
- 3.9 Apply perfect secrecy, all-or-nothing secret sharing, and solving lights out games to vectors over GF(2). LACS.GSR.3.9
- 3.10 Use vector operations to program simple authentication schemes. LACS.GSR.3.10

5 Solve contextual, mathematical problems involving matrices as geometric transformations and to explain real-life phenomena. LACS.GSR.5

- 5.1 Given a 2-by-2 or 3-by-3 linear transformation matrix, describe the transformation a geometric figure undergoes. LACS.GSR.5.1
- 5.2 Find matrices that represent scalings, reflections, and rotations of geometric figures. LACS.GSR.5.2
- 5.3 Find a matrix that represents a combination of transformations. LACS.GSR.5.3
- 5.4 Find the image of a point under a transformation. LACS.GSR.5.4
- 5.5 Find the area of a polygon given its coordinates using matrices; find the area of the image of a polygon after a transformation. LACS.GSR.5.5
- 5.6 Write code to perform transformations in two-dimensional geometry using matrix operations. LACS.GSR.5.6
- 5.7 Define functions from n dimensions to m dimensions as vectors and/or matrices. LACS.GSR.5.7
- 5.8 Find the image and preimage of a linear map using matrices; determine whether the linear map is one-to-one. LACS.GSR.5.8
- 5.9 Find and interpret geometrically the set of preimages of a vector under a given matrix. LACS.GSR.5.9

Patterning & Algebraic Reasoning

4 Solve contextual, mathematical problems involving matrices to explain real-life phenomena. LACS.PAR.4

- 4.1 Represent a linear system of three equations in three variables as an augmented matrix and reduce the matrix to row-echelon form. LACS.PAR.4.1
- 4.2 Interpret the nature of the solution of a system from its row-echelon form, and if there are infinitely many solutions, express them as a vector equation. LACS.PAR.4.2
- 4.3 Determine whether a vector is a linear combination of other given vectors; find the linear combination of vectors that results in a given vector. LACS.PAR.4.3
- 4.4 Interpret linear dependence of vectors geometrically. LACS.PAR.4.4
- 4.5 Find the kernel of a matrix and explore the relationship between the kernel, the orthogonality of the vectors in the kernel, and the linear dependence of the rows/columns. LACS.PAR.4.5
- 4.6 Add two matrices, multiply a matrix by a scalar, find the transpose of a matrix. LACS.PAR.4.6
- 4.7 Determine when matrix multiplication is defined, and if defined, multiply two matrices by considering the matrix product as a dot product of a group of vectors. LACS.PAR.4.7
- 4.8 Determine when the inverse of a square matrix exists, and if it exists, find it by augmenting the identity matrix to the matrix and then use row operations. LACS.PAR.4.8
- 4.9 Decompose a matrix into its symmetric and skew-symmetric parts; decompose a matrix into its LU factorization. LACS.PAR.4.9
- 4.10 Solve a matrix equation using inverses; find all solutions to a matrix equation given one solution and the kernel. LACS.PAR.4.10
- 4.11 Improve the simple authentication scheme over GF(2). LACS.PAR.4.11
- 4.12 Show and explain how threshold secret sharing works in conjunction with Gaussian elimination through programming. LACS.PAR.4.12
- 4.13 Write code utilizing error-correcting concepts. LACS.PAR.4.13

7 Solve contextual, mathematical problems using vector spaces to explain real-life phenomena. LACS.PAR.7

- 7.1 Determine whether a given set of vectors generates a vector space. LACS.PAR.7.1
- 7.2 Justify whether a subset of a vector space is a subspace. LACS.PAR.7.2
- 7.3 Determine whether a given vector is in the linear span of a set of vectors. LACS.PAR.7.3
- 7.4 Determine whether two vector subspaces are orthogonal; find the orthogonal component of a given subspace. LACS.PAR.7.4
- 7.5 Determine whether a set of vectors is a basis for a vector space. LACS.PAR.7.5
- 7.6 Find the dimension of a vector space; find the dimensions of the row space, column space, and kernel for a given matrix; find the rank of a matrix. LACS.PAR.7.6
- 7.7 Find a matrix representing a linear map. LACS.PAR.7.7
- 7.8 Determine the change of representation for a linear transformation given two different bases on a vector space. LACS.PAR.7.8
- 7.9 Determine if two matrices are similar; determine if two matrices are orthogonal. LACS.PAR.7.9
- 7.10 Find an orthogonal basis for a given basis or subspace by applying the Gram-Schmidt orthonormalization process. LACS.PAR.7.10
- 7.11 Perform QR factorization of a matrix to solve matrix equations. LACS.PAR.7.11
- 7.12 Apply the method of least squares to find the line or parabola of best fit to approximate data in context. LACS.PAR.7.12
- 7.13 Apply the grow-and-shrink algorithm in the minimum spanning forest problem in GF(2). LACS.PAR.7.13
- 7.14 Apply the Exchange Lemma to image perspective rendering. LACS.PAR.7.14
- 7.15 Use bases to represent images and sounds as wavelets; perform wavelet transformation, implementation, and decomposition through programming. LACS.PAR.7.15
- 7.16 Program a Fast Fourier Transform to store a sequence of amplitude samples. LACS.PAR.7.16
- 7.17 Apply the Rank Theorem to demonstrate the simple authentication scheme. LACS.PAR.7.17

8 Solve contextual, mathematical problems using eigenvalues and eigenvectors to explain real-life phenomena. LACS.PAR.8

- 8.1 Evaluate the determinant of a matrix along any row or column and use a recursive procedure for evaluating a determinant for matrices larger than 3-by-3. LACS.PAR.8.1
- 8.2 Justify properties of the determinant. LACS.PAR.8.2
- 8.3 Calculate the determinant of the product of two matrices; calculate the determinant of the transpose of a matrix. LACS.PAR.8.3
- 8.4 Determine if a matrix has a nonzero determinant and extend the nonzero determinant property to problems involving linear dependency, rank, and matrix inverses. LACS.PAR.8.4
- 8.5 Extend the definition and geometric interpretation of the cross product to n – 1 vectors in n dimensions. LACS.PAR.8.5
- 8.6 Use Cramer's Rule to solve a system of linear equations. LACS.PAR.8.6
- 8.7 Find the characteristic polynomial of a matrix and interpret the characteristic polynomial geometrically. LACS.PAR.8.7
- 8.8 Find the eigenvalues and eigenvectors of a matrix and interpret them geometrically. LACS.PAR.8.8
- 8.9 Use a basis of eigenvectors to create a change of basis matrix. LACS.PAR.8.9
- 8.10 Find the dimension of the eigenspace corresponding to the eigenvalues of a symmetric matrix. LACS.PAR.8.10
- 8.11 Determine an orthogonal matrix that diagonalizes a given matrix. LACS.PAR.8.11
- 8.12 Apply eigenvalues and eigenvectors to problems in context. LACS.PAR.8.12

Probabilistic Reasoning

6 Using probabilistic and quantitative reasoning, solve contextual, mathematical problems using Markov chains to explain real-life phenomena. LACS.PR.6

- 6.1 Model a finite random process using transition matrices in a Markov chain. LACS.PR.6.1
- 6.2 Simulate the different stages of a Markov chain using random numbers. LACS.PR.6.2
- 6.3 Use matrix algebra to calculate the probability of future states of a Markov chain. LACS.PR.6.3
- 6.4 Determine the attractor for a regular Markov chain. LACS.PR.6.4
- 6.5 Use transition matrices to identify absorbing states of a Markov chain. LACS.PR.6.5
- 6.6 Apply Markov chains in context. LACS.PR.6.6
- 6.7 Write a program to model the probabilities of real-life phenomena using a Markov chain. LACS.PR.6.7