

# Advanced Technical Mathematics (2021): Grades 9, 10, 11, 12

Adopted 2021

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them. [MP.1](#)

---

2. Reason abstractly and quantitatively. [MP.2](#)

---

3. Construct viable arguments and critique the reasoning of others. [MP.3](#)

---

4. Model with mathematics. [MP.4](#)

---

5. Use appropriate tools strategically. [MP.5](#)

---

6. Attend to precision. [MP.6](#)

---

7. Look for and make use of structure. [MP.7](#)

---

8. Look for and express regularity in repeated reasoning. [MP.8](#)

## The Real Number System

1. Solve real-world and mathematical problems involving the four operations with rational numbers. [RN.1](#)

---

2. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. [RN.2](#)

---

3. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. [RN.3](#)

---

4. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. [RN.4](#)

---

5. Represent proportional relationships by equations. [RN.5](#)

---

6. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate. [RN.6](#)

---

**7. Use proportional relationships to solve multistep ratio and percent problems.** [RN.7](#)

---

**Measurement and Data**

**8. Recognize volume as an attribute of solid figures and understand concepts of volume measurement wherein a cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.** [MD.8](#)

---

**9. Recognize volume as an attribute of solid figures and understand concepts of volume measurement wherein a solid figure which can be packed without gaps or overlaps using  $n$  unit cubes is said to have a volume of  $n$  cubic units.** [MD.9](#)

---

**10. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.** [MD.10](#)

---

**11. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.** [MD.11](#)

---

**12. Apply the formulas  $V = l \times w \times h$  and  $V = b \times h$  for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.** [MD.12](#)

---

**13. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.** [MD.13](#)

---

**14. Use units as a way to understand problems and to guide the solution of multi-step problems, choose and interpret units consistently in formulas, choose and interpret the scale and the origin in graphs and data displays.** [MD.14](#)

---

**Equations, Expressions, and Functions**

**15. Write and evaluate numerical expressions involving whole-number exponents.** [EE.15](#)

---

**16. Write expressions that record operations with numbers and with letters standing for numbers.** [EE.16](#)

---

**17. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.** [EE.17](#)

---

**18. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order.** [EE.18](#)

---

- 
- 19.** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set. [EE.19](#)
- 
- 20.** Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. [EE.20](#)
- 
- 21.** Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. [EE.21](#)
- 
- 22.** Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. [EE.22](#)
- 
- 23.** Know and apply the properties of integer exponents to generate equivalent numerical expressions. [EE.23](#)
- 
- 24.** Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. [EE.24](#)
- 
- 25.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers). [EE.25](#)
- 
- 26.** Solve linear equations and inequalities with rational number coefficients, including those whose solutions require expanding expressions using the distributive property and collecting like terms. [EE.26](#)
- 
- 27.** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. [EE.27](#)
- 
- 28.** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [EE.28](#)

- 
- 29.** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. **EE.29**
- 
- 30.** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **EE.30**
- 
- 31.** Write a function that describes a relationship between two quantities; and be able to calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **EE.31**
- 
- 32.** Construct linear functions, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). **EE.32**
- 

## Geometry and Spatial Reasoning

- 33.** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. **G.33**
- 
- 34.** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **G.34**
- 
- 35.** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **G.35**
- 
- 36.** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. **G.36**
- 
- 37.** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. **G.37**
- 
- 38.** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. **G.38**
- 
- 39.** Explain a proof of the Pythagorean Theorem and its converse. **G.39**
- 
- 40.** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **G.40**

- 
- 41. Explain and use the relationship between the sine and cosine of complementary angles.** G.41
- 
- 42. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.** G.42
- 
- 43. Identify and describe relationships among inscribed angles, radii, and chords.** G.43
- 
- 44. Construct the inscribed and circumscribed circles of a triangle; and, prove properties of angles for a quadrilateral inscribed in a circle.** G.44
- 
- 45. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.** G.45
- 
- 46. Use coordinates to prove simple geometric theorems algebraically.** G.46
- 
- 47. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.** G.47